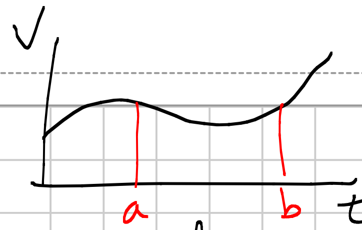
Note Title 2013-09-23

Calc 12 - Chp 7.1



Integral as Net Change

Vector Quantity – a measurement that has direction. This means that it can be positive or negative.
Displacement is a vector between the initial and final points.

eg) Force, displacement, velocity, acceleration, momentum, impulse

Scalar Quantity – a measurement without direction. This means it is always a positive value.
Distance is the absolute value of displacement.

eg) volume, area, mass, time, charge, temperature, distance, energy, work, power, length, speed, mass, density

Recall: Area Under the Curve – is $\int |f(x)|\,dx$. So the answer for any scalar quantity is $\int |f(x)|\,dx$
To use definite integrals, you need to find the zeroes of $f(x)$.

eg)

In Exercises 1–8, the function $v(t)$ is the velocity in m/sec of a particle moving along the x-axis. Use analytic methods to do each of the following:

(a) Determine when the particle is moving to the right, to the left, and stopped.

(b) Find the particle's displacement for the given time interval.

(c) Find the total distance traveled by the particle.

#2 $v(t) = 6 \sin 3t, \quad 0 \le t \le \pi/2$

a) $v(t) = 0 = 6 \sin 3t$ $\qquad b = 3$

b) $s(x) = \int_0^x 6\sin 3t\,dt$ displacement.

c) distance $= \int_0^{\pi/3} 6\sin 3t\,dt - \int_{\pi/3}^{\pi/2} 6\sin 3t\,dt$

31. *Hooke's Law* Hooke's Law also applies to *compressing* springs; that is, it requires a force of kx to compress a spring a distance x from its natural length. Suppose a 10,000-lb force compressed a spring from its natural length of 12 inches to a length of 11 inches. How much work was done in compressing the spring

(a) the first half-inch? **(b)** the second half-inch?

$W(y) = \int_0^y |kx|\,dx$

a) $F = k|x|$

②

b) $w(1/2) =$

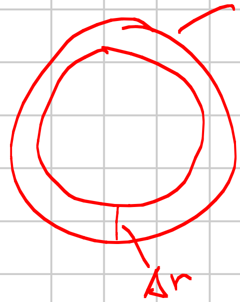
24. *Oil Flow* Oil flows through a cylindrical pipe of radius 3 inches, but friction from the pipe slows the flow toward the outer edge. The speed at which the oil flows at a distance r inches from the center is $8(10 - r^2)$ inches per second.

(a) In a plane cross section of the pipe, a thin ring with thickness Δr at a distance r inches from the center approximates a rectangular strip when you straighten it out. What is the area of the strip (and hence the approximate area of the ring)?

(b) Explain why we know that oil passes through this ring at approximately $8(10 - r^2)(2\pi r)\,\Delta r$ cubic inches per second.

(c) Set up and evaluate a definite integral that will give the rate (in cubic inches per second) at which oil is flowing through the pipe.

a)



$A \sim 2\pi r \Delta r$

b) $\frac{dV}{dt}(r) =$

c) $\int_0^3 \frac{dV}{dt}(r)\,dr =$

Note: our lower limit of integration is usually zero, but use the interval given.

Hw: pp. 371-372: 1, 3, 7, 9, 11, 17, 19, 21, 29, 20
Challenge: 23, 31

Calc 12 - Chp 7.2

Areas in the Plane

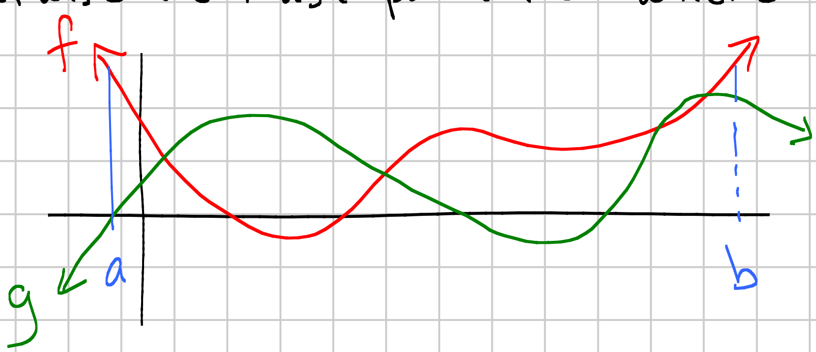
Area Between the Curves
Area Enclosed by Intersecting Curves
Integrating wrt y

Area Between the Curves with interval

$A = \int_a^b |f(x) - g(x)|\,dx$

If $f(x) \geq g(x)$ on $[a, b]$ then $A = \int_a^b f(x) - g(x)\,dx$
Otherwise we must partition where $f(x) - g(x) = 0$

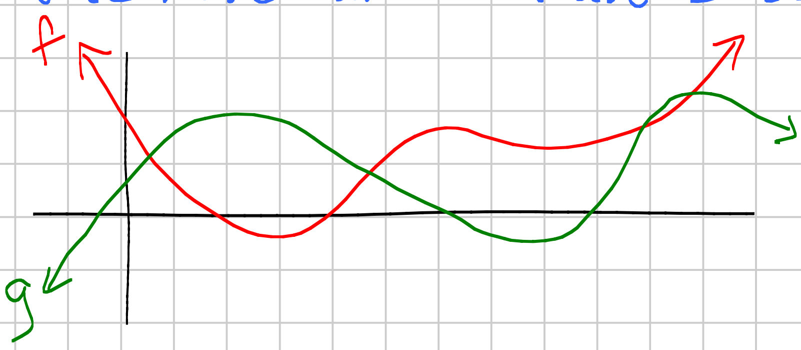
eg)



Area Enclosed by Intersecting Curves (no interval)

In this case, $[a, b]$ is not given. So solve $f(x) - g(x) = 0$. a = smallest zero and b = largest zero.

Note must have at least 2 zeroes.



Integrating wrt 'y'

Some regions are easier to solve by having x as a function of y; ie relations. Solve equations in terms of y; also convert boundaries in terms of y.

eg) Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

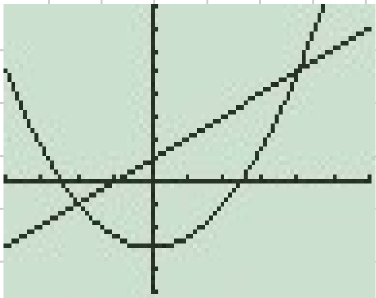
$y = x - 1$

$y^2 = 2x + 6$

$X_L =$

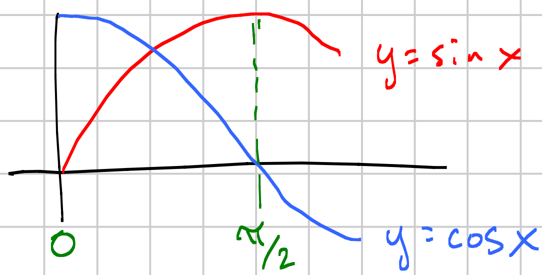
$X_R =$

$A =$



eg) Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi/2$.

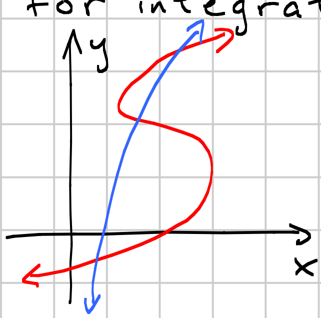
Sketch:



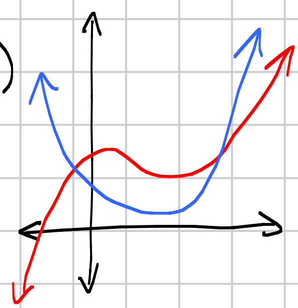
$\sin x = \cos x$

Look for functions to determine which variable to use for integration.

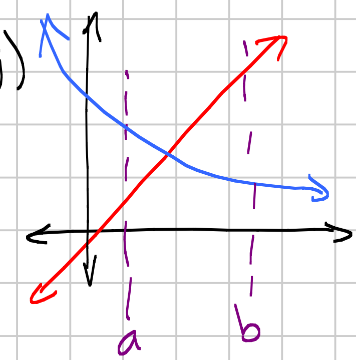
eg)



eg)



eg)



HW: pp. 380-382: 1-7 odds, 11, 13, 19, 21, 27, 20

Challenge: 43

Calc 12 - Chp 7.3 Part 1

Volumes

$\underbrace{\text{Volumes as Integrals, Cross Sections}}_{\text{Part 1}}$ includes discs

$\underbrace{\text{Washer Method, Cylindrical Shells}}_{\text{Part 2}}$

Recall: $\text{Area} = \sum_{i=1}^{n} f(x_i)\Delta x$

Similarly: $\text{Volume} = \sum_{i=1}^{n} \text{Area}(x_i)\Delta x$

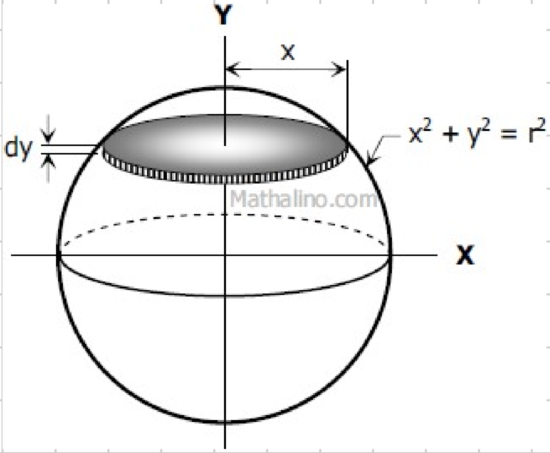
as $\lim_{\Delta x \to 0}$ the volume will be exact.

Definition: Volume of a Solid - the volume of a solid of known integrable cross section area $A(x)$ from $x=a$ to $x=b$ is:

Notes:

- The cross section area $A(x)$ must be a consistent shape as x changes, ie. $A(x)$ is a single formula.
- Some setups will be easier to solve.
- Can use symmetry to multiply integral of part of the object.

eg) Volume of a sphere $V = \int_{-r}^{r} A(y)\,dy$



We could have solve by calculating the volume of a hemisphere and doubling the answer.

$\frac{1}{2}V = \int_0^r \pi (r^2 - y^2)\, dy$

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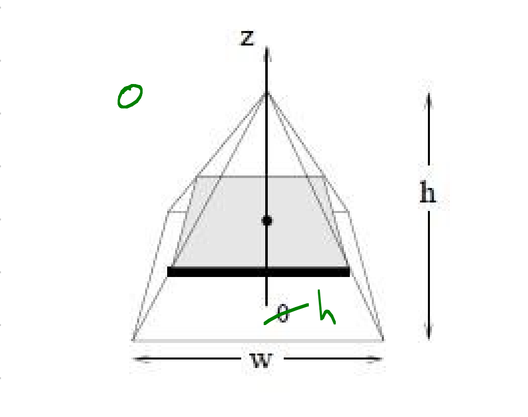
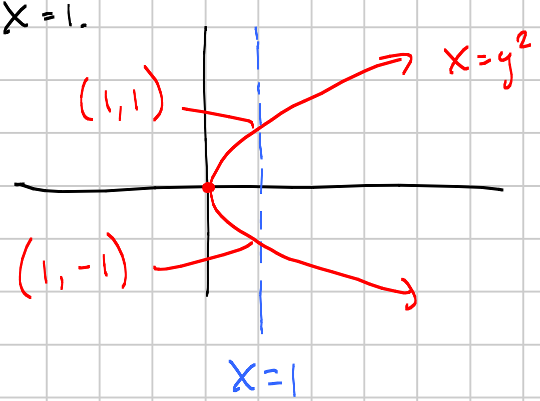
eg) Area of a square base pyramid



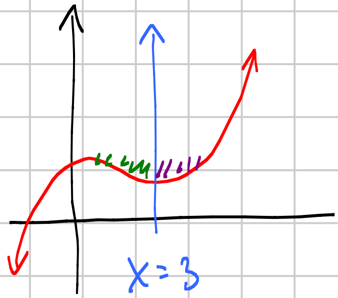
Rotation of a Function about a Line - make a sketch so that you get the correct radius. This creates disks. If rotating about x, integrate wrt y. If rotating about y, integrate wrt x. Axis of rotation should be one of the bounds.

eg) Find the volume of the solid by rotating the region bounded by $x = y^2$ and $x = 1$ about $x = 1$.



The disk method works when you have a function and integrating lines are perpendicular to AoR. It does not work for a relation.

eg)



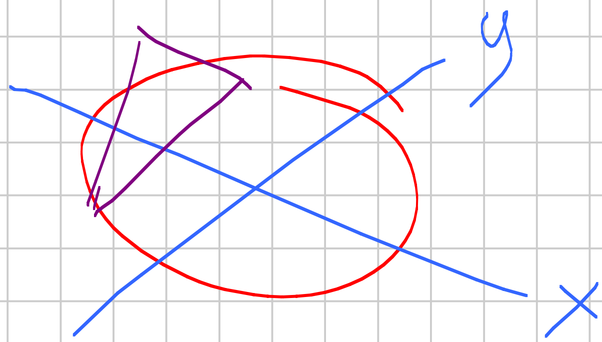
HW: pp. 390-392: 1, 13-19 odd.

Volume of a cone with height 'h' and base radius of 'r'

Challenge: 11

Question 1 is having you get used to creating functions.

1c) Area of square is dependent on 'x'



Calc 12 - Chp 7.3 Part 2

Note Title 2013-09-23

Volumes

Volumes as Integrals
$\underbrace{\text{Cross Sections}}_{\text{Part 1}}$

Washer Method
$\underbrace{\text{Cylindrical Shells}}_{\text{Part 2}}$

eg) Washer Method - find the volume of the solid obtained by rotating the region bounded by $y=x$ and $y=x^3$ about the line $y=2$ with $x \geq 0$.

$x^3 = x \Rightarrow$

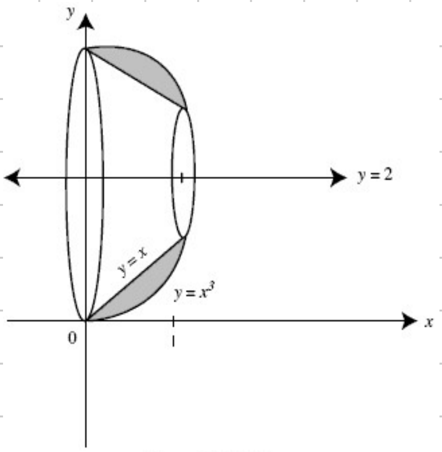
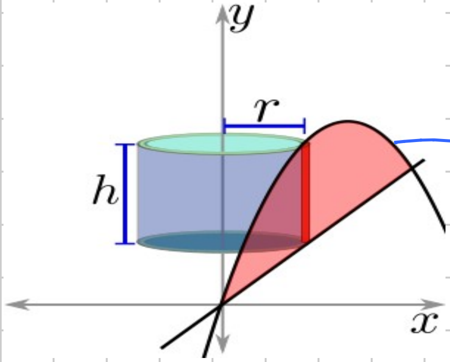


Figure 12.4-13

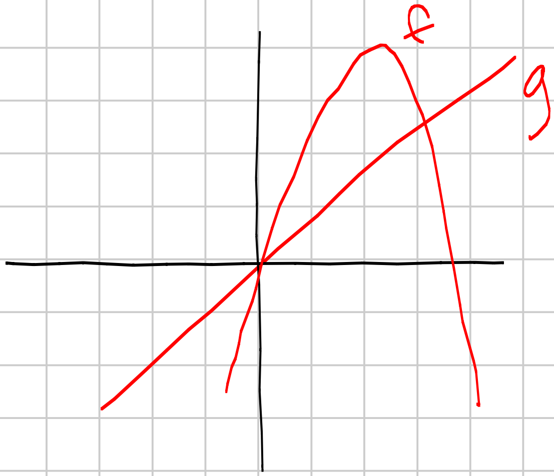
The washer method works when you have 2 functions and the integrating lines are perpendicular to AoR.

Cylindrical Shells - sometimes washers won't work because there won't be a single function to represent the washers. Sometimes there is a relation instead of a function in the direction of the integration.



washer is a small problem here because there are two x values at this y-value.

eg) Find the volume of the solid obtained by rotating the region bounded by $y = -(x-2)^2 + 4$ and $y = x$ about the y-axis.



The shell method works when you have functions and the integrating lines are parallel to AoR.

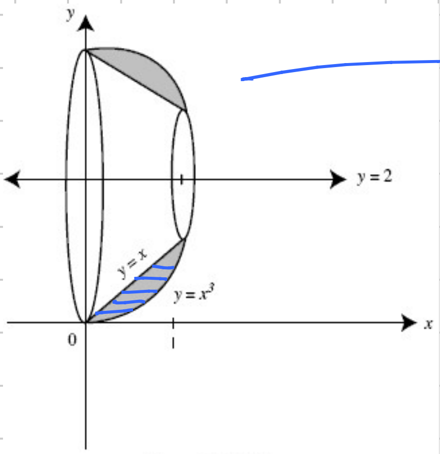
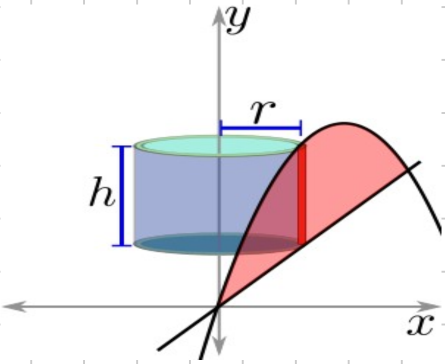


Figure 12.4-13

This could be set up as cylindrical shells as well.



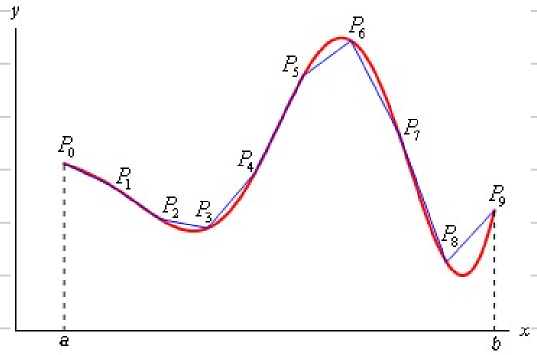
Hw: pp. 392-393: 21, 23, 25, 39, 41, 43
Set up the integral for $x = 1 - y^4$, $x = 0$ about $x = 2$. DON'T SOLVE!

Challenge: 35, 53

Calc 12 - Chp 7.4

Note Title 2013-09-23

Lengths of Curves AKA arclength



If f' is continuous on [a,b], then arc length:

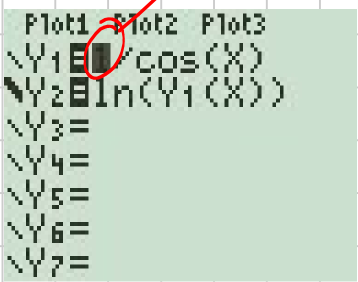
$$L = \int_a^b$$

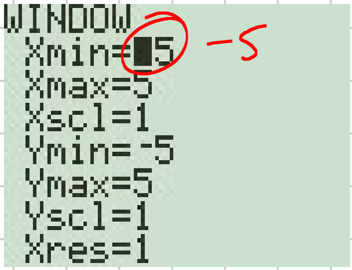
Unfortunately, the $\sqrt{\quad}$ makes integration difficult or impossible to do analytically, so it is usually solved numerically.

eg) Find the arc length of $y = x^2$ from $x = 0$ to $x = 1$

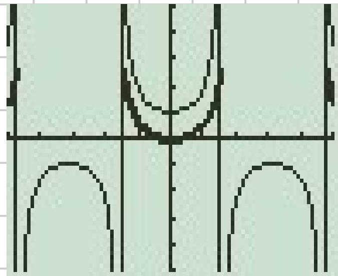
eg) Find the arc length of $y = \ln(\sec x)$, $0 \le x \le \pi/4$

1



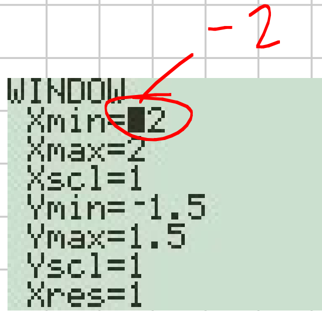


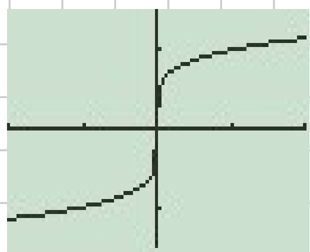
−5



If $f'(x)$ is not continuous, it may be that $f^{-1}(y)$ is continuous, so: (such as vertical tangent)

eg) Find the arclength of $y = \sqrt[5]{x}$ between $x = -1$ and $x = 1$.





HW: p. 399: 1-15 odds, 2
Challenge: 17

Calc 12 - Chp 7.5

Applications - Work / Pressure / Probabilities.

Skip

Work is force times distance:

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \int_a^b f(x)\,dx$$

As long as force is in the same direction, we can sum up the separate work.

Simple eg) A force of 40 N is needed to hold a spring stretched from its natural length of 10 cm to a length of 15 cm. How much work is done stretching it another 3 cm? Hooke's Law: $f(x) = kx$, where x is the length beyond the natural length.

Med eg) A heavy rope, 50 ft long, weighs ½ lb/ft and hangs over a 100 ft high building.

a) How much work is done pulling the rope to the top of the building.

b) How much work is done pulling half the rope up.

a)

b)

Hard eg) A 2kg bucket and a 20 m - 5 kg rope are used to haul water from a 20 m deep well. The bucket holds 20 kg of water. The bucket is pulled up a rate of .5 m/s. Water leaks out a rate of .1 kg/s. How much work is done bringing the bucket to the top?

Bucket =

Rope =

Water :

lbs (pounds) is a unit of force not mass!

Imperial mass are slugs, slinches, & snails

HW: pp. 407- 408: 1 - 11 odds

A cable that is 3kg/m is used to lift 200kg of coal up a 300 m mineshaft. Find the work done.

Challenge: 33, 35