Calc 12 - Chp 3 Review

Note Title 2013-09-23

Definition of Derivative: (slope)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Derivative D.N.E. if there is a discontinuity, corner, or vertical tangent.

Derivative Rules (with Chain Rule)

$$\frac{d}{dx}(c) = 0 \qquad \frac{d}{dx}(u^n) = n u^{n-1} u' \qquad \frac{d}{dx}(cu) = c u'$$

$$\frac{d}{dx}(u \pm v) = u' \pm v' \qquad \frac{d}{dx}(uv) = uv' + u'v$$

$$\frac{d}{dx}\frac{u}{v} = \frac{u'v - uv'}{v^2} \qquad \frac{d}{dx}\sqrt{u} = \frac{1}{2\sqrt{u}} u'$$

$$\frac{d}{dx}(\sin u) = (\cos u) u' \qquad \frac{d}{dx}(\cos u) = (-\sin u) u'$$

$$\frac{d}{dx}(\tan u) = (\sec^2 u) u' \qquad \frac{d}{dx}(\csc u) = (-\csc u)(\cot u) u'$$

$$\frac{d}{dx}(\sec u) = (\sec u)(\tan u) u' \qquad \frac{d}{dx}(\cot u) = (-\csc^2 u) u'$$

$$\frac{d}{dx}(\sin^{-1} u) = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}(\cos^{-1} u) = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}(\tan^{-1} u) = \frac{u'}{1+u^2} \qquad \frac{d}{dx}(\csc^{-1} u) = \frac{-u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}(\sec^{-1} u) = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}(\cot^{-1} u) = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}(e^u) = (e^u) u' \qquad \frac{d}{dx}(a^u) = (a^u)(\ln a) u'$$

$$\frac{d}{dx}(\ln u) = \frac{u'}{u} \qquad \frac{d}{dx}(\log_a u) = \frac{u'}{u(\ln a)}$$

Implicit Differentiation:

1. Differentiate both sides wrt x.
2. Collect y' terms on LHS, others on RHS.
3. Factor out y'.
4. Isolate y'.

Second Derivative Implicit:

1. Differentiate RHS.
2. Substitute expression of y'

Reciprocal and Quotient Identities.

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta \sin\theta = 1 \qquad \sec\theta \cos\theta = 1 \qquad \cot\theta \tan\theta = 1$$

Pythagorean Identities.

$$\cos^2\theta + \sin^2\theta = 1$$
$$\sin^2\theta = 1 - \cos^2\theta$$
$$\cos^2\theta = 1 - \sin^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$
$$\tan^2\theta = \sec^2\theta - 1$$
$$\sec^2\theta - \tan^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta$$
$$\cot^2\theta = \csc^2\theta - 1$$
$$\csc^2\theta - \cot^2\theta = 1$$