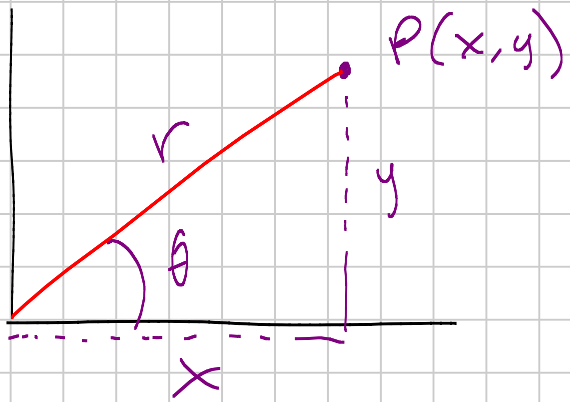
PreCalc 11 Chp 6.1

Note Title 2015-12-10

Angles in Standard Position in Quadrant 1



Mnemonics

Syr : $\sin\theta = \frac{y}{r}$

Cxr : $\cos\theta = \frac{x}{r}$

Tyx : $\tan\theta = \frac{y}{x}$

eg) Determine Trig Ratios if P(5,6) is on the terminal arm in standard position.

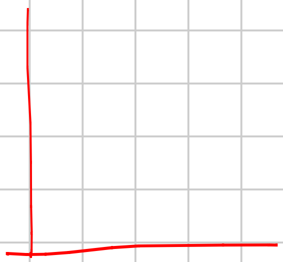
Draw diagram

Radians are the measure of arc length to radius.

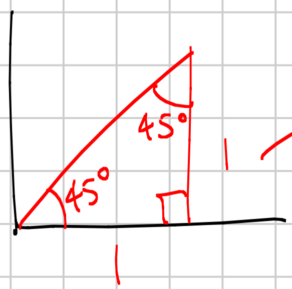
Conversion radians = $\frac{\deg(\pi)}{180}$ deg = $\frac{180 \text{ rads}}{\pi}$

eg) A boat travels 45 km from St. John's Newfoundland at a heading of N15°E to go fishing. How far north and east did the boat travel?

Draw



45° ($\pi/4$ radians) is a special angle.



Pythagorean Formula: $\sqrt{a^2+b^2} = c$

$\sin 45° = \cos 45° =$

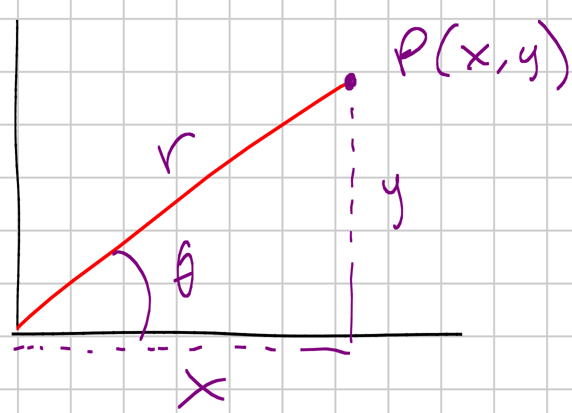
$\sin \pi/4 = \cos \pi/4 =$

A simple rearrangement of SYR, CXR, TYX:

$\sin \theta = \frac{y}{r} \Rightarrow r \sin \theta = y$

$\cos \theta = \frac{x}{r} \Rightarrow r \cos \theta = x$

$\tan \theta = \frac{y}{x}$ = slope.



HW: pp. 431-438: 3-5, 7, 9-11, 12
Challenge: 14-16 (answer all in radians)
Please show 8 i in person

PreCalc 11 Chp 6.2

Note Title 2015-12-16

Angles in Standard Position in All Quadrants

Because of reflections, there is symmetry in the trig ratios.

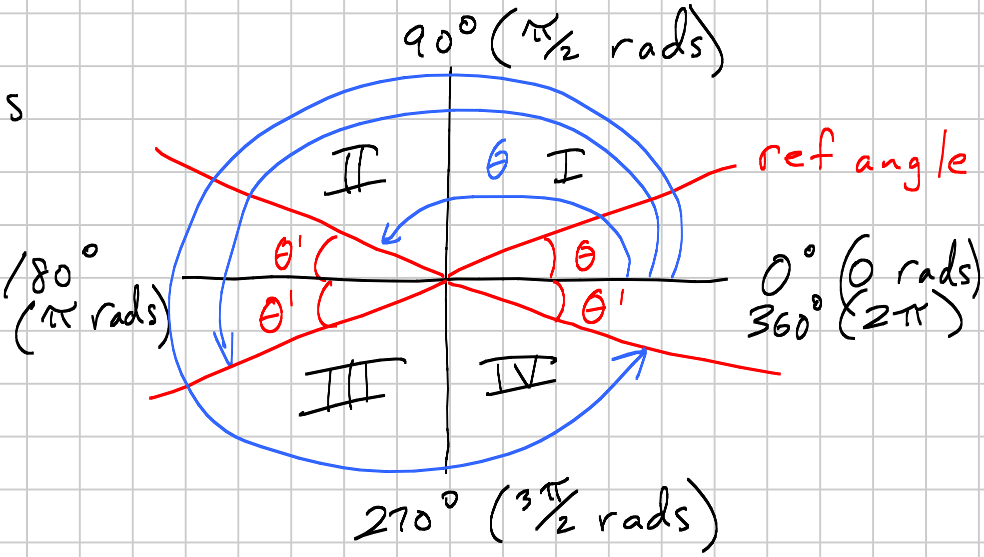
Reference Angles
(converting to)

I: $\theta' =$

II: $\theta' =$

III: $\theta' =$

IV: $\theta' =$



Standard Angles (Converting to)

I: $\theta =$

II: $\theta =$

III: $\theta =$

IV: $\theta =$

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eg) Convert to reference angles:

a) 300°

b) 100°

c) 220°

d) 70°

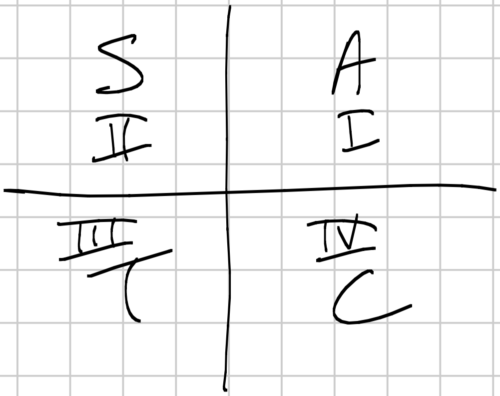
eg) Convert to standard angles:

a) 40° in II

b) 35° in IV

c) 65° in I

d) 15° in III



All Students Take Calculus.

All: All trig ratios +'ve

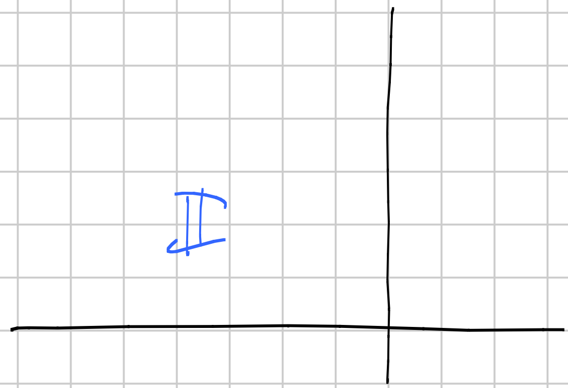
Students: $\sin\theta$ is +'ve, $\cos\theta$, $\tan\theta$

Take: $\tan\theta$ is +'ve, $\sin\theta$, $\cos\theta$

Calculus: $\cos\theta$ is +'ve, $\sin\theta$, $\tan\theta$ are -'ve

This mnemonic states the positive trig ratios, all others are negative.

eg) $P(-3, 7)$ lies on the terminal arm with θ in standard position. Determine the primary trig ratios and θ.



Definition: Principal Angle is the coterminal angle such that $0 \leq \theta < 360°$.

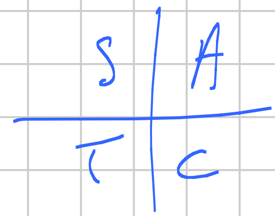
eg) P.A. for $-30°$ is

P.A. for $800°$ is

P.A. for $200°$ is

The calculator will produce answers for two quadrants, so you must understand when and how to adjust you solution (angles).

eg) Determine the Principal Angle for θ such that $\sin\theta = -.6$



eg) Given $\cos\theta = -5/13$, determine the other primary trig ratios and the Principal Angles.

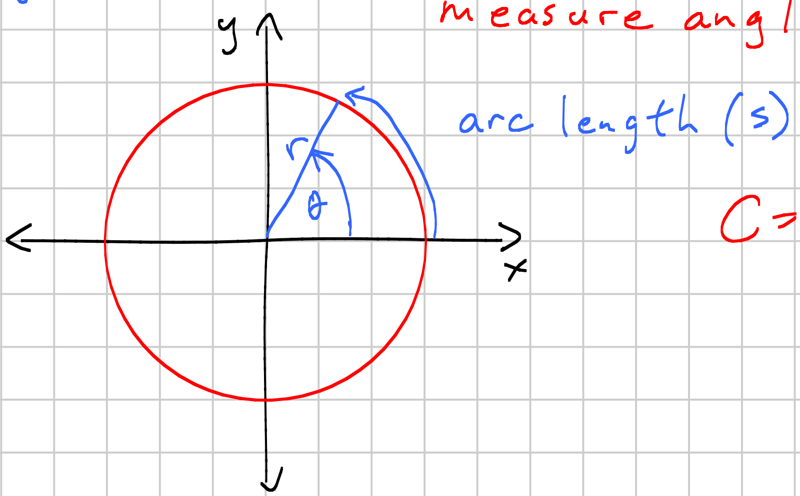
HW: pp. 448-458: 3-7, 10, 11, 13-15, 17a, 18a
Challenge: 12, 17b, 18b, 19 (answer all in radians)
Please show 9 in person.

Pre Calc 11 - Chp 6.3

Note Title 2015-12-17

Radian Measure

Definition: Radian is the ratio between the arc length and the radius. This is another way to measure angle!

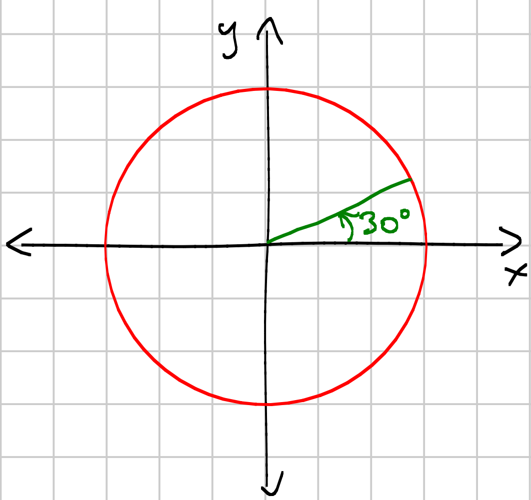


$C = 2\pi r$

eg) convert to radians:

$30^\circ =$ $\qquad\qquad$ $270^\circ =$

$-135^\circ =$ $\qquad\qquad$ $720^\circ =$



Angles in Standard Position start with the initial side along the positive x-axis. It rotates counter-clockwise until the terminal side is reached. (positive dir) We can freely rotate multiple times CW or CCW to reach the terminal side.

Every time we go all the way around the circle CCW, we add 360° to the angle. So this would be once around the ferris wheel in the regular direction.

Every time we go all the way around the circle CW, we subtract 360° from the angle. So this would be once around the ferris wheel in the opposite direction.

Any time two angles have the same terminal side, we say they are coterminal. eg)

EVERY angle has an infinite number of coterminal sides! We write this as:

Find 2 more coterminal angles of 30°.

Special Angles:

Degrees	Radians	sin	cos	tan
0	0	0	1	0
30	$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
60	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90	$\pi/2$	1	0	undef
180	π	0	-1	0
270	$3\pi/2$	-1	0	undef
360	2π	0	1	0

These are not difficult to remember as long as you can use Pythorean to figure them out. Remember that 30°, 45°, 60° are reflected on both axes into the other 3 quadrants. When you come across these special angles, you are expected to give the exact ratio instead of decimal approximations.

eg) State the primary trig ratios for 210°.

HW:

1. As a fraction of π, determine ~~the length of the arc that subtends each central angle in the unit circle~~. the radian measure.

a) $180°$

b) $135°$

c) $150°$

d) $-45°$

4. Sketch each angle in standard position.

a) $\frac{\pi}{4}$

b) $\frac{\pi}{2}$

c) $\frac{5\pi}{3}$

d) $\frac{3\pi}{4}$

e) $-\frac{\pi}{6}$

f) $-\frac{3\pi}{4}$

5. a) Convert each angle to radians.

i) $60°$ **ii)** $720°$ **iii)** $-450°$

b) Convert each angle to degrees. Give the answer to the nearest degree where necessary.

i) 7π **ii)** 5 **iii)** $-\frac{5\pi}{6}$

5. For each angle in standard position below:

i) Determine the measures of angles that are coterminal with the angle in the given domain.

ii) Write an expression for the measures of all the angles that are coterminal with the angle in standard position.

iii) write all answers in radians

a) $75°$; for $-500° \leq \theta \leq 500°$

b) $-105°$; for $-600° \leq \theta \leq 600°$

c) $215°$; for $-700° \leq \theta \leq 700°$

d) $-290°$; for $-800° \leq \theta \leq 800°$

ANSWERS

1. **a)** π **b)** $\frac{3}{4}\pi$ **c)** $\frac{5}{6}\pi$ **d)** $\frac{1}{4}\pi$

4. **a)** approximately: 2.5, −3.8 **b)** approximately: 2.7, 5.8, −3.6, −0.5

5. **a)** **i)** $\frac{\pi}{3}$ **ii)** 4π **iii)** $-\frac{5\pi}{2}$ **b)** **i)** 1260° **ii)** approximately 286° **iii)** −150°

5. **a)** **i)** 75°, 435°, −285°
ii) $75° + k360°, k \in \mathbb{Z}$ **b)** **i)** 255°, −105°, −465° **ii)** $\cancel{-105°}$ 255° $+ k360°, k \in \mathbb{Z}$
c) **i)** 215°, 575°, −145°, −505° **ii)** $215° + k360°, k \in \mathbb{Z}$
d) **i)** 70°, 430°, 790°, −290°, −650° **ii)** $\cancel{-290°}$ 70° $+ k360°, k \in \mathbb{Z}$

a) iii) $5\pi/12$, $29\pi/12$, $-19\pi/12$, $5\pi/12 + 2\pi k,\ k \in \mathbb{Z}$

b) iii) $17\pi/12$, $-7\pi/12$, $-31\pi/12$, $17\pi/12 + 2\pi k,\ k \in \mathbb{Z}$

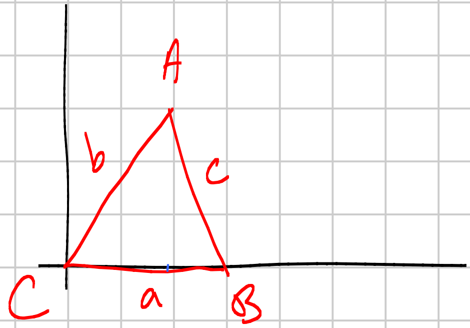
c) iii) $43\pi/36$, $115\pi/36$, $-29\pi/36$, $-101\pi/36$, $43\pi/36 + 2\pi k,\ k \in \mathbb{Z}$

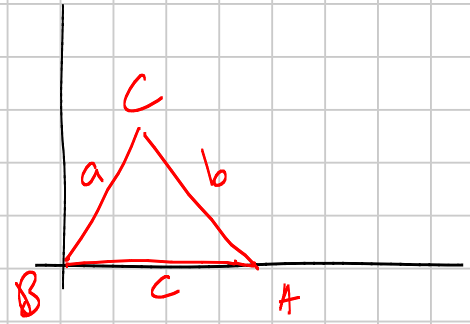
d) iii) $7\pi/18$, $43\pi/18$, $79\pi/18$, $-29\pi/18$, $-65\pi/18$, $7\pi/18 + 2\pi k,\ k \in \mathbb{Z}$

Pre Calc 11 - Chp 6.4

Note Title 2015-12-17

The Sine Law (derivation)





We can see that

Likewise

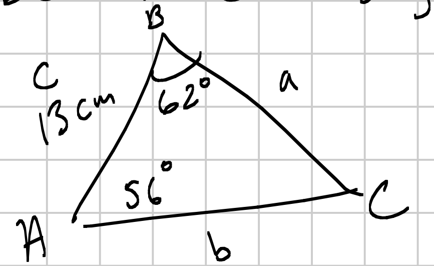
And using transitivity: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$!

or $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

This is an important property since not all triangles are right triangles. Please understand the derivation as this is the best way to remember the formula.

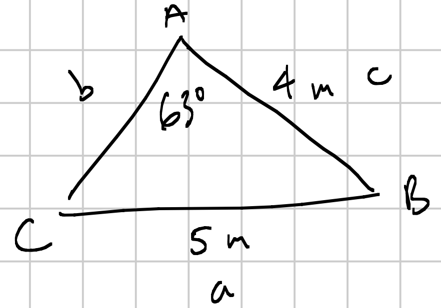
eg) Case: ASA (angle-side-angle)

Determine all angles and sides:



eg) Case : ASS

Determine all angles and sides



It is possible that ASS is not triangle :

$A = 30°$, $b = 10$ m, $a = 2$ m

It is also possible that ASS gives you 2 triangle solutions : $A = 30°$, $b = 10$ m, $a = 6$ m (if no diagram)

Rule for no solution ASS:

- $a < c \sin A$ can substitute eg) $b < a \sin B$

Rule for 2 triangles (ambiguous) case ASS:

- $A < 90°$ if $A = 90°$ then it is just a right triangle
- $a < c$
- $a > c \sin A$

eg) Are the following ambiguous:

$B = 60°$, $a = 20$ m, $b = 5$ m

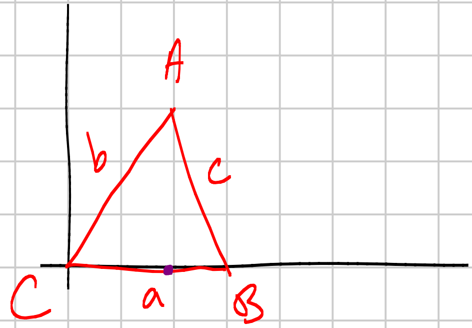
$B = 60°$, $a = 10$ m, $b = 9$ m

$C = 50°$, $a = 9$ m, $c = 12$ m

$C = 40°$, $a = 13$ m, $C = 11$ m

HW: pp. 478-489: 3-6, 10, 13, 14
Challenge: 15-17
Please show 7ai in person.

The Cosine Law (Derivation for SAS or SSS)



This time we are give a, b, C

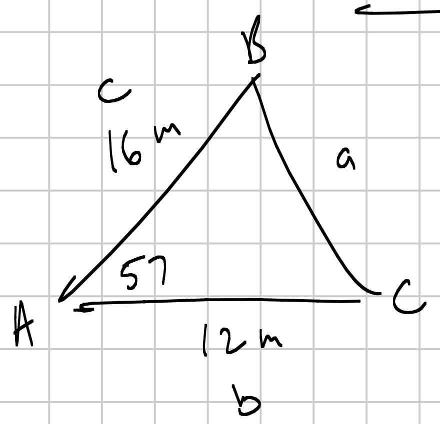
This looks like Pythagorean and an extra term: $-2ab\cos C$
Since the triangle is arbitrarily drawn, we also have

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$

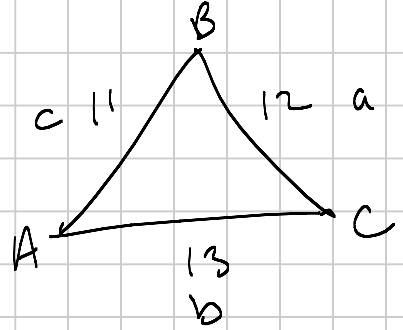
There are no ambiguous cases for the cosine law.
How does this relate to Pythagorean? If c is the hypotenuse, then $C = 90°$ and $\cos 90° = 0$, so $-2ab\cos C = 0$.
Then we have $c^2 = a^2 + b^2$ and it's the same!

Strategy: Use cosine law once, then use sine law for the remaining unknowns. Why?

eg)



eg)



Definition: Bearing - does not match angles in standard position. North is defined as 0° and incrementing is clockwise, so east is 90°, south is 180°, and west is 270°.

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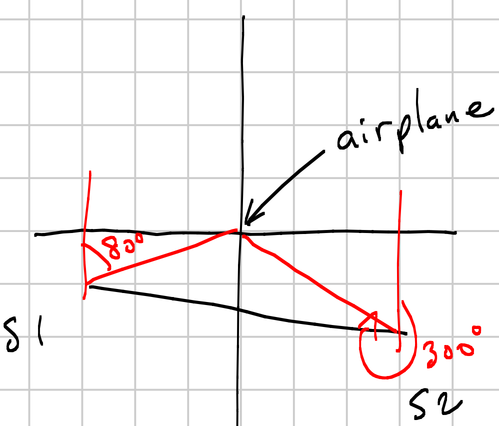
eg) Find the distance between two ships. One ship spots an airplane at a bearing of 80° and an elevation of 7°. The other ship spots the plane with a bearing of 300° and an elevation of 11°. The airplane reportes its altitude as 10,000 m Report your answer in km.



HW: pp. 498-506: 5-7, 9, 10, 12
Challenge: 11, 13, 14
Please show 3 in person.