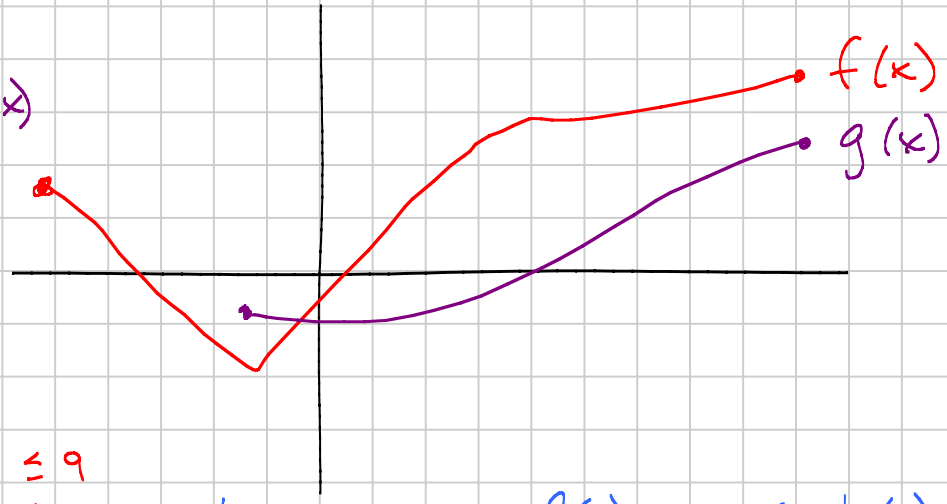
PreCalc 12 Chp 4.1

Note Title 2013-09-29

Combining Functions Graphically
Adding, subtracting, multiplying, & dividing.

Adding:

$h(x) = f(x) + g(x)$

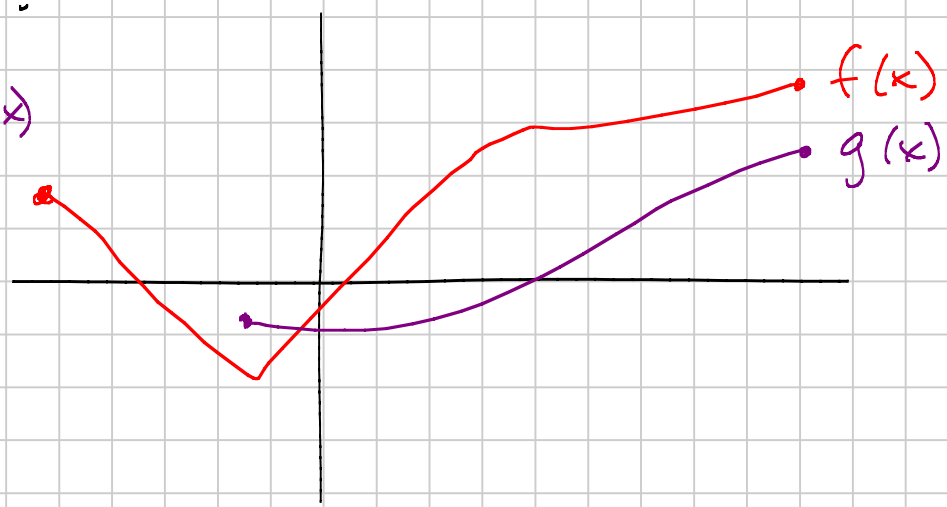


D: $-5.2 \le x \le 9$
R: $-2 \le y \le 3.6$
D: $-1.5 \le x \le 9$
R: $-1 \le y \le 2.4$
D:
R:

Key points: $f(x) = 0 \Rightarrow h(x) = g(x)$
$g(x) = 0 \Rightarrow h(x) = f(x)$
$f(x) = -g(x) \Rightarrow h(x) = 0$
endpoints

Subtracting:

$h(x) = f(x) - g(x)$



D:
R:

Key points: $f(x) = 0 \Rightarrow h(x) = -g(x)$
$g(x) = 0 \Rightarrow h(x) = f(x)$
$f(x) = g(x) \Rightarrow h(x) = 0$
endpoints

Multiplying:

$h(x) = f(x) \cdot g(x)$

D:

R:

Key points:

$f(x) = 0 \Rightarrow h(x) = 0$

$g(x) = 0 \Rightarrow h(x) = 0$

$f(x) = 1 \Rightarrow h(x) = g(x)$

$g(x) = 1 \Rightarrow h(x) = f(x)$

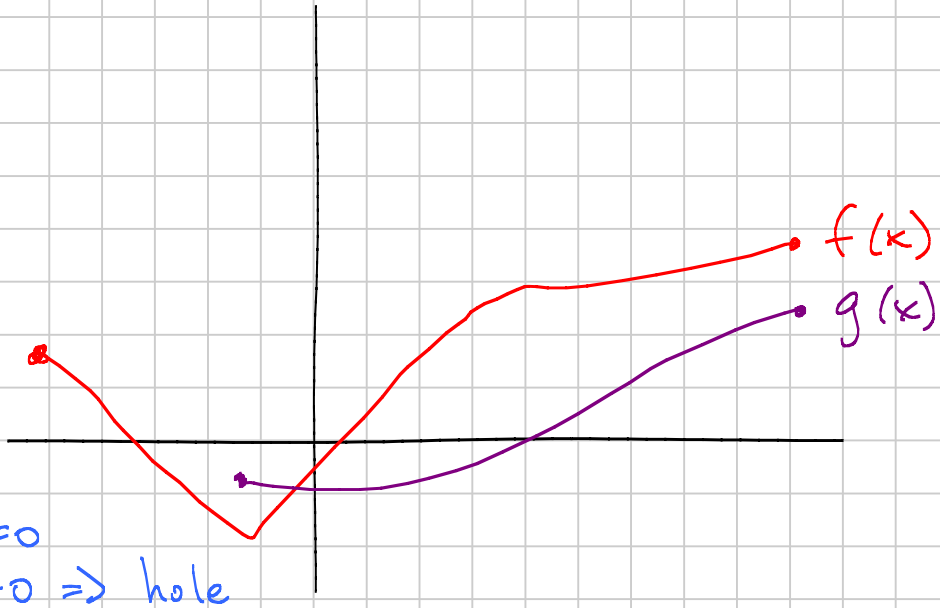
$f(x) = -1 \Rightarrow h(x) = -g(x)$

$g(x) = -1 \Rightarrow h(x) = -f(x)$

endpoints

Dividing:

$h(x) = \frac{f(x)}{g(x)}$



D:

R:

Key points:

$f(x) = 0 \Rightarrow h(x) = 0$

$g(x) = 0$ & $f(x) = 0 \Rightarrow$ hole

$g(x) = 0$ & $f(x) \neq 0 \Rightarrow$ V.A.

$g(x) = 1 \Rightarrow h(x) = f(x)$

$g(x) = -1 \Rightarrow h(x) = -f(x)$

$f(x) = g(x) \Rightarrow h(x) = 1$

$f(x) = -g(x) \Rightarrow h(x) = -1$

endpoints

In general, it is difficult to say anything about the combined range because the max and min of one function may not correspond to the max and min of the other function.

However, we can make some generalizations for the combined domain:

- it is the most restrictive domain; in other words, where the 2 domains overlap
  - eg) D of f: $-2 \le x \le 3$
    D of g: $-5 \le x \le 1$
    D of f+g: $-2 \le x \le 1$
- for division, there are NPV's when $g(x) = 0$. This is where there will be a VA or a hole.

(FYI: you have to know that a term will cancel out. 0/0 is a clue; so watch out for this on Q 1d!)

Alternative to key points is to use a table of values; however, you may miss the NPV's.

HW: pp. 268-271: 1, 2, 3a, 3b
Challenge 3c
Please solve 3d in person.

PreCalc 12 Chp 4.2

Note Title 2013-09-29

Combining Functions Algebraically
Add, Subtract, Multiply, and Divide.

Big Thinking:

Add & Subtract will behave like most dominant term.

eg) $f(x) = x^3 + x$, $g(x) = x^2 + \sqrt{x}$

$h(x) = f(x) \pm g(x)$ will behave like

Multiply will behave like product of dominant terms.

eg) $h(x) = f(x)\,g(x)$ will behave like

Divide will behave like quotient of dominant terms.

eg) $h(x) = f(x)/g(x)$ will behave like

Function Notation Revisited

Definition: $f(x) = x^4 + x^3 + \sqrt{-x} - 5$

Evaluating: $f(-6) =$

$f(4-7) =$

$f(y^2-5) =$

Reminder: Use more restrictive domain!

Adding: $h(x) = f(x) + g(x) = (f+g)(x)$

eg) $f(x) = x^3 + x$, $g(x) = x^2 + \sqrt{x}$

D of f: $\mathbb{R}$ R of f: $\mathbb{R}$ D of g: $x \geq 0$ R of g: $y \geq 0$

$h(x) =$

Subtracting $h(x) = f(x) - g(x) = (f-g)(x)$

$h(x) =$

Multiplying: $h(x) = f(x)\,g(x) = (f \cdot g)(x)$

$h(x) =$

Dividing: $h(x) = f(x)/g(x) = (f/g)(x)$

$h(x) =$

Finding Functions - usually many solutions, pick EASY ones!

eg) $k(x) = x^4 - 3x^3 + 2x^2 + 10$. Find f, g, & h such that $k(x) = f(x) + g(x) - h(x)$

eg) $h(x) = x^2 - 5x + 6$. Find f & g such that $h(x) = (f \cdot g)(x)$

eg) $h(x) = x^3$. Find f & g such that $h(x) = \frac{f(x)}{g(x)}$

Be careful of domain!

Avoid trivial solutions, you may lose marks! Don't pick $f(x) = 0$ or $g(x) = 0$ for addition or subtraction. Don't pick $f(x) = c$ or $g(x) = c$ for multiplication or division.

HW: pp. 278-284: 3, 4, 6, 7, 8, 10, 12, 14, 16
Challenge 15, 18
Please solve 13 in person

PreCalc 12 Chp 4.3

Note Title 2013-09-29

Intro to Composite fns – take the output of one fn and use it as an input to another fn. This can happen more than once.

This is common in business.

$f(g(x)) = (f \circ g)(x)$
"f of g of x"

x = Cost (p)
y = DistributionPrice (x)
z = WholesalePrice (y)
u = RetailPrice (z)
v = Tax (u)
$v = T(R(W(D(C(p)))))$
$= (T \circ R \circ W \circ D \circ C)(p)$

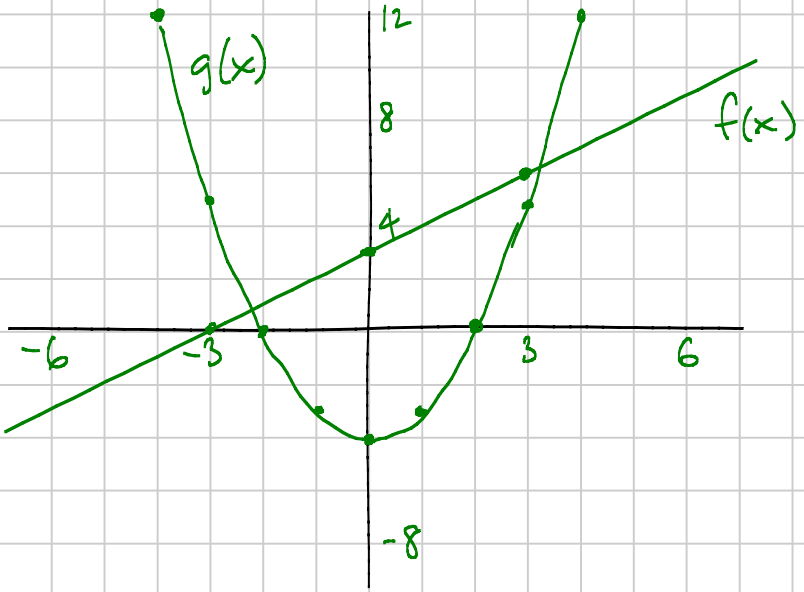
Generally: $f(g(x)) \neq g(f(x))$

Example: $f(x) = x + 3$ $g(x) = x^2 - 4$

$f(g(1)) = ?$

$g(f(1)) = ?$

$f(g(x)) =$ $g(f(x)) =$



Using tables:

x	f(x)
-2	0
-1	1
0	2
1	-1
2	-2

x	g(x)
-2	1
-1	2
0	-2
1	-1
2	0

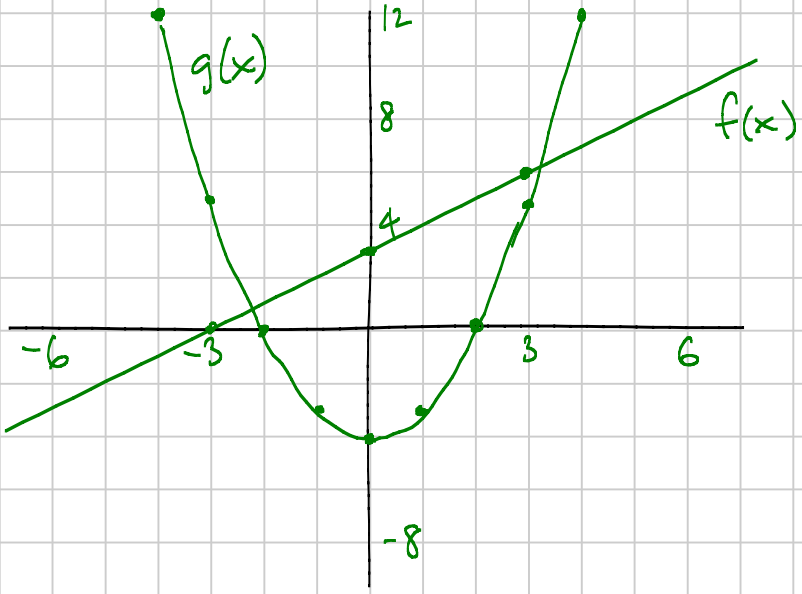
a) $f(g(-2))$

b) $g(f(-2))$

c) $f(f(-1))$

d) $g(f(0))$

Backwards Problem



a) $f(g(a)) = 4$
What is a?

b) $g(f(a)) = 12$
What is a?

Linear Inverse Fns Revisited.
If $f(g(x)) = x$, then f & g are inverses of each other. Substitute using brackets. Although only one test is needed, do both to verify your algebra is correct.

eg) $f(x) = 4x - 9$ $\quad g(x) = \frac{x+9}{4}$

$f(g(x)) =$

$g(f(x)) =$

eg) $f(x) = 3x - 7$ $\quad g(x) = \frac{x+6}{2}$

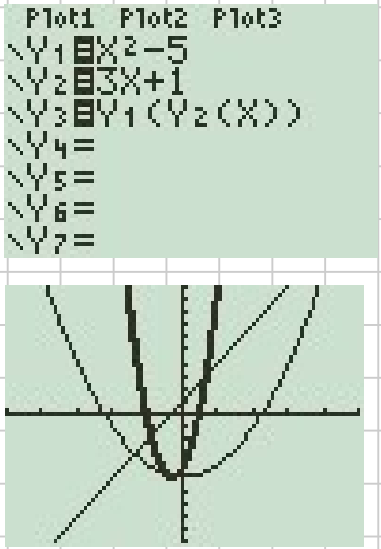
$f(g(x)) =$

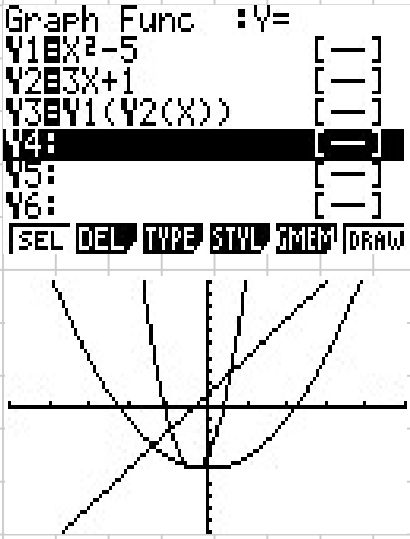
$g(f(x)) =$

Using the calculator for composite fns.

11) $f(x) = x^2 - 5$ $\quad g(x) = 3x + 1$

$f(g(x))$





Harder Question (similar to 9c p. 300)

eg) $f(x) = |3 - x|$, $g(x) = (x-6)^2$, $h(x) = \sqrt{x}$

Determine $f(g(h(5)))$.

HW: pp. 298-303: 4, 5, 6, 8, 9, 10, 12, 14, 16

Challenge: 15, 17

Please show 13a in person.

PreCalc 12 Chp 4.4

Note Title 2013-09-29

Determining Restrictions on Composite fns.
This is very different from the restrictions on combining fns and more difficult - recall that we take the most restrictive domain.

eg) $f(x) = \sqrt{x}$ $g(x) = x^2 + 2$

Determine domain of $f(g(x))$ & $g(f(x))$

D of f: R of f:

D of g: R of g:

$f(g(x)) = \sqrt{x^2+2}$

$\therefore$ D of $(f \circ g)$: R of $(f \circ g)$:

$g(f(x)) = (\sqrt{x})^2 + 2 = x + 2$

D of $(g \circ f)$: R of $(g \circ f)$:

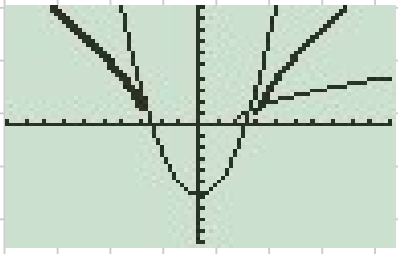
You can reason it out or you can explicitly express the composition to figure it out.

If it's not obvious, use your calculator and look for min/max for the range.

eg) $f(x) = \sqrt{2x-4}$ $g(x) = x^2 - 6$

Determine domain and range of $(f \circ g)(x)$.

$f(g(x)) = \sqrt{2(x^2-6)-4}$



↑ hard even with calculator

because you can't set the bounds properly.

There are 2 types of questions: determine an explicit equation for the composition and determine possible equations to create the composition. The latter is the more difficult task.

eg) Determine the possible equations for the composite fn $h(x) = \sqrt{x^2-3}$. Find f & g.

What strategies can you use?

- BEDMAS - whatever steps you use for an evaluation are good candidates for your inner functions.
- For polynomials, completing the square gives you an inner function, use this or risk losing marks!
- Remember - combining fns is not composition

eg) $k(x) = \sqrt{x^2-4x+7}$ Find f, g, h, j.

eg) $m(x) = \dfrac{1}{3x^2 + 6x - 15}$ Find f, g, h, j, k

eg) Different Strategy Case

$h(x) = (-x+4)^3 + (-x+4)^2 + 7.$ Find f & g.

Look for pattern!

HW: pp. 314-320: 3, 4, 6, 7, 8, 10, 11
Challenge: 12, 13
Please show 9 in person.